UNIT-II

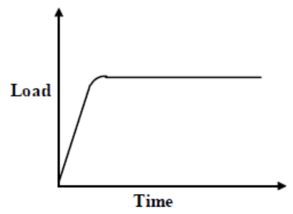
STRENGTH OF MACHINE ELEMENTS

We have discussed, in the previous chapter, the stresses due to static loading only. But only a few machine parts are subjected to static loading. Since many of the machine parts (such as axles, shafts, crankshafts, connecting rods, springs, pinion teeth etc.) are subjected to variable or alternating loads (also known as fluctuating or fatigue loads), therefore we shall discuss, in this chapter, the variable or alternating stresses.

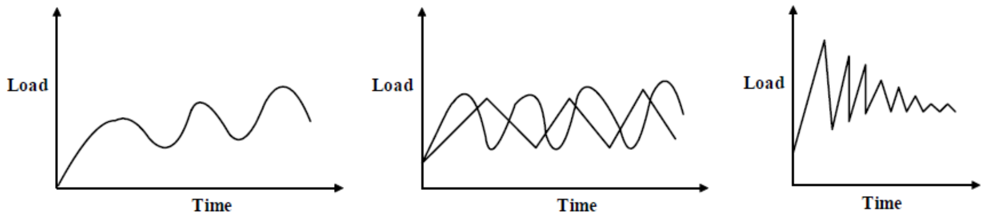
a) Static load- Load does not change in magnitude and direction and normally increases gradually to a steady value.

b) Dynamic load- Load may change in magnitude for example, traffic of varying weight passing a bridge. Load may change in direction.

Example, load on piston rod of a double acting cylinder.



Static loading



Dynamic loading

Introduction:-

Elementary equations for stresses,

Tensile stress $\sigma_t = \frac{P}{A}$

Bending stress = $\sigma_b = \frac{M\ y}{I}$

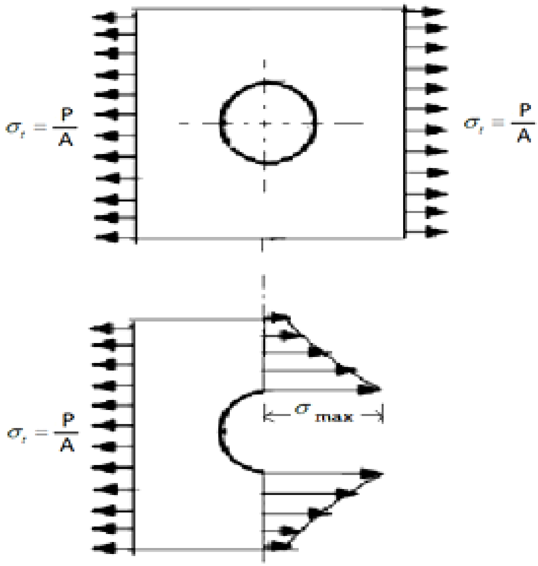
Shear stress = $\tau = \frac{T\ .\ r}{J}$

The above equations are based on assumptions, that there are no discontinuities in the cross section of the component.

A plate, with a small circular hole subjected to tensile stress is shown in figure.

It is observed from the nature of stress distribution at the section passing through the hole, that there is a sudden rise in the magnitude of stresses in the neighborhood of the hole.

The localised stresses in the neighborhood of the hole are far greater than the stresses obtained by the elementary equations.



Stress Concentration

<u>Stress Concentration</u> (Define theoretical stress concentration factor)

Stress concentration (or) theoretical stress concentration is defined as the localisation of high stresses due to irregularities (or) abrupt changes in the cross-section of a component.

$$K_t = \frac{\text{Heighest value of actual stress near discontinuity}}{\text{Normal stress obtained by elementary equations for minimum cross-section}}$$

$$K_t = \frac{\sigma_{max}}{\sigma_o} = \frac{\tau_{max}}{\tau_o}$$

Where K_t = Theoritical stress concentration factor

σ_{max} = Max. Normal stress, τ_{max} = Max. shear stress

σ_o = Normal Nominal stress, τ_o = normal Shear stress

σ_o and τ_o are the stresses calculated by the elementary equations for minimum cross-section,

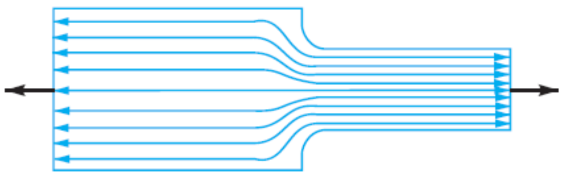
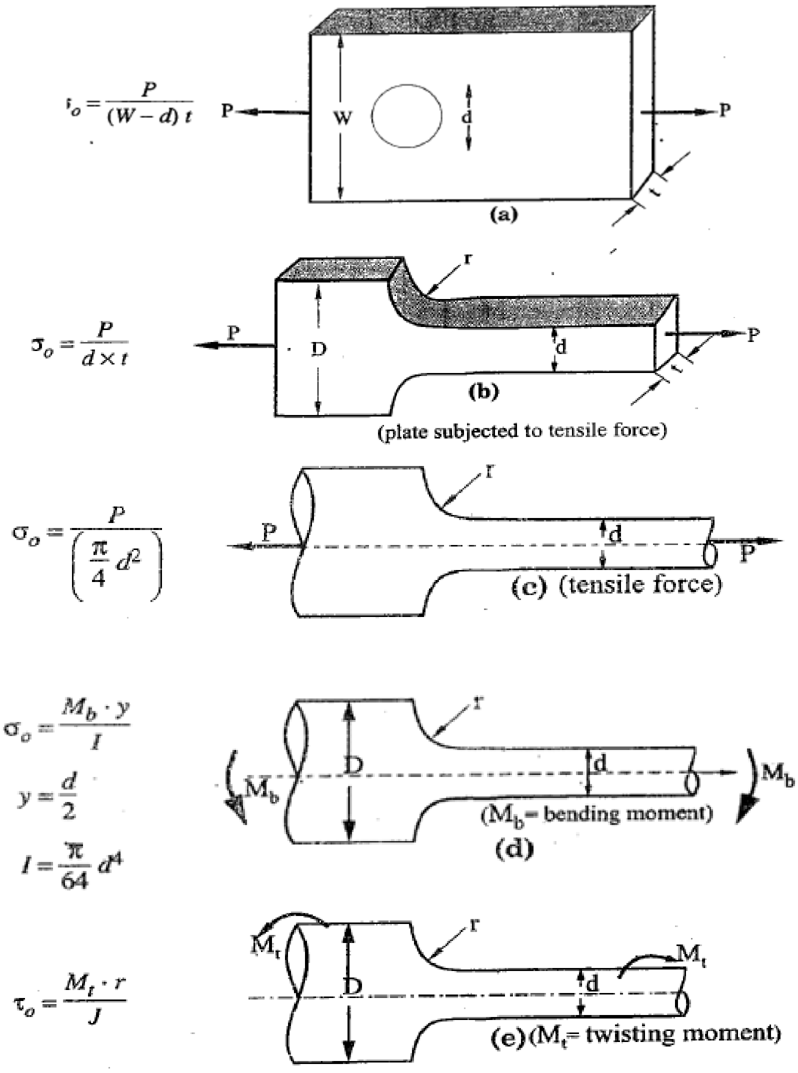


Fig: Stress concentration

NOMINAL STRESS:-



Causes of stress concentration:-(Explain briefly about the causes of stress concentration)

1. Sudden changes in cross section:- Abrupt changes in the cross section of components like steps and shoulders on the shafts induce stress concentration at these cross sections.
2. Irregularities or discontinuities: - Presence of discontinuities like oil holes or grooves, keyways, splines and screw threads, which cause stress concentration.
3. Indentation or Scratches:- Indentation like stamp marks or inspection marks and machining scratches are surface irregularities, which cause stress concentration.
4. Variation in material properties:- Variation in properties material induces stress concentration in the material. This variation due to the presence of,

Foreign or non-metallic inclusions, Internal cracks, Cavities, Blow holes.

5. Application of load:- If the applied load is concentrated over a very small area, then it results in stress concentration. Some of the example are, Wheels on a rolling surface, Cam and follower pair, Ball bearing and Meshing gears etc.

<u>Method to determine stress concentration factors</u>:- (Describe the method to determine stress concentration factors)

The stress concentration factors are determined by two methods.

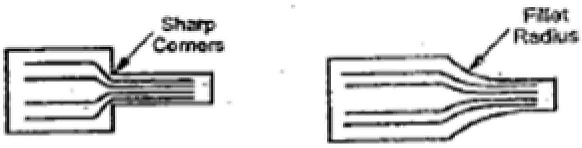
1. Mathematical analysis based on the theory of elasticity,
2. Experimental methods like Photo-elasticity

These are limitations for the techniques of the theory of elasticity, For more complex shapes, the stress concentration factors are determined by photo-elasticity. static loads, ductile materials are not affected by stress concentration Under

Under static loads, brittle materials are affected by stress concentration.

<u>Methods of reducing stress concentration</u>:-(Explain the methods of reducing stress concentration)

1. <u>By providing large fillet radius</u>:-



Poor Preferred

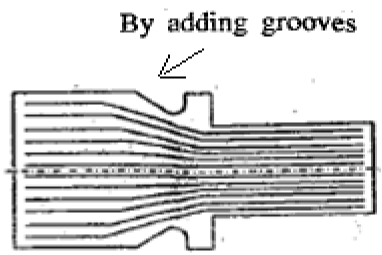
Providing large fillet radius at sharp corners makes the cross sectional area to decrease more gradually instead of suddenly. This gradual decrease in cross section distributes the stress more uniformly in the body and reduces stress concentration.

2. <u>By drilling small holes</u>:-

Poor Preferred

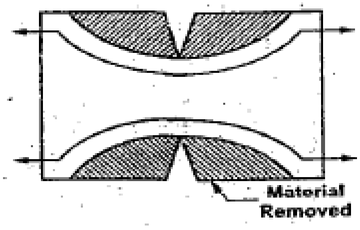
Small holes are drilled near the big holes to distribute the stress more uniformly. This is because; the intensity of stress near the bigger hole is more. Therefore, to reduce the stress concentration near the bigger hole, smaller holes are provided in the direction of the load.

3. <u>By adding Grooves</u>:-



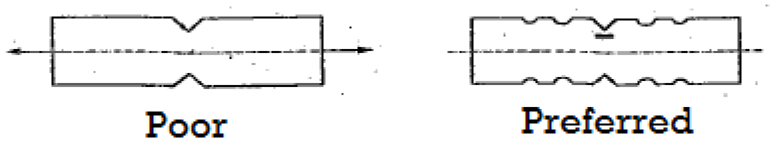
Stress relieving grooves are provided to distribute the stress more uniformly.

4. By removing undesired material:-



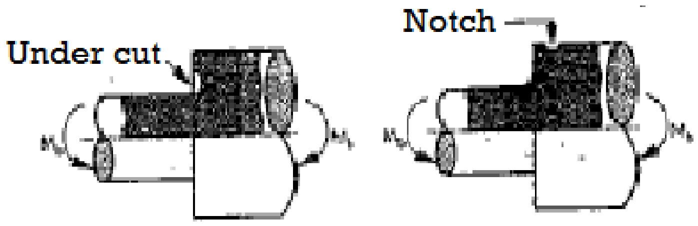
Stress concentration can be reduced by removing the undesired material from the component. The method of removing undesired material is called the principle of minimization of the material.

5. By providing additional notch:-



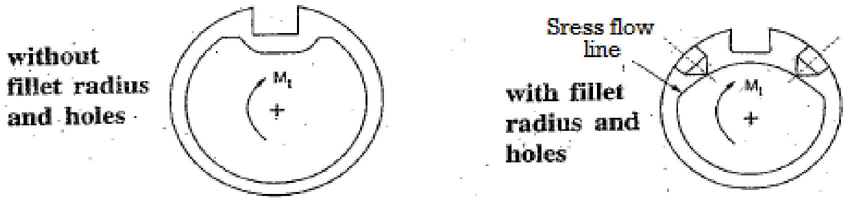
Usually, a notch results in stress concentration. However, cutting an additional notch nearer to the original notch reduces the stress concentration.

6. By providing additional notch and undercutting:-



In a circular bar provided with shoulder, the shoulder creates a change in cross-section of the shaft, which results in stress concentration. This stress concentration can be reduced either by cutting an additional notch or by increasing the fillet radius by undercutting the shoulder.

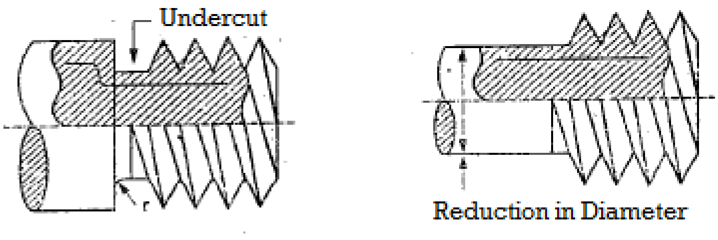
7. Reduction of stress concentration in shaft with keyway:-



Stress concentration in shaft with keyway can be reduced by two methods.

- By providing a fillet radius at the inner corners of the keyway: This makes the stress to distribute more uniformly and reduces the stress concentration.
- By drilling two symmetrical holes at the sides of the keyway: These holes reduces bending in the vicinity of the keyway.

8. Reduction of stress concentration in threaded members:-



Stress concentration in threaded members can be reduced by two methods.

- By providing a undercut between the shank and the threaded portion: This reduces bending of the force flow line and reduces the stress concentration.
- By reducing the shank diameter equal to the core diameter: This will make the force flow line almost straight and there is no stress concentration.

Fatigue Stress Concentration Factor

When a machine member is subjected to cyclic or fatigue loading, the value of fatigue stress concentration factor shall be applied instead of theoretical stress concentration factor.

"It is defined as the ratio of endurance limit without stress concentration to the endurance limit with stress concentration"

$$K_f = \frac{\text{Endurance limit without stress concentration}}{\text{Endurance limit with stress concentration}}$$

(OR)

$$K_f = \frac{\text{Endurance limit of a notched specimen}}{\text{Endurance limit of a notch-free specimen}}$$

(OR)

$$K_f = \frac{\text{Maximum stress in notched specimen}}{\text{stress in notch-free specimen}}$$

Notch Sensitivity(Define the notch sensitivity)

The notch sensitivity of a material is a measure of how sensitive a material is to notches of geometric discontinuities.

factor may be obtained from the following relations:

$$q = \frac{K_f - 1}{K_t - 1}$$

The values of q are between zero and unity. It is evident that

- If q = 0, then $K_f = 1$, the material has no sensitivity to notches at all
- If q = 1, then $K_f = K_t$, and the material has full notch sensitivity.

$$K_f = 1 + q\,(K_t - 1) \quad \text{...[For tensile or bending stress]}$$

$$K_{fs} = 1 + q\,(K_{ts} - 1) \quad \text{...[For shear stress]}$$

Where K_t = Theoretical stress concentration factor for axial or bending loading,
K_{ts} = Theoretical stress concentration factor for torsional or shear loading.

Problem(1):-Determine the maximum stress produced in a rectangular plate 50mm wide, 8 mm thick with a central hole of 10mm diameter. It is loaded in an axial tension of 1 KN. Assume stress concentration factor K_t =2.5

Given data:

Width $= W = 50$ mm

Thickness $= h = 8$ mm

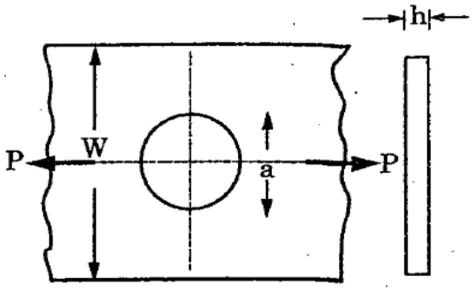
Dia. of centre hole $= a = 10$ mm

Axial tension $= P = 1$ kN $= 1 \times 10^3$ N

Stress concentration $= K_t = \frac{\sigma_{max}}{\sigma_o}$ **To find K_t**

K_t = Stress concentration

σ_{max} = Maximum stress = N/mm²



$$\sigma_0 = \text{Nominal stress} = \frac{P}{(W-a)\,h}$$

$$= \frac{1 \times 10^3}{(50-10)\,8} = 3.125 \text{ N/mm}^2$$

$$\therefore K_t = \frac{\sigma_{max}}{\sigma_0},$$

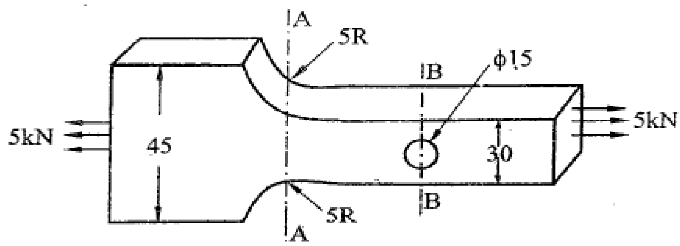
$$2.5 \times 3.125 = \sigma_{max},$$

The maximum stress $= \sigma_{max} = 7.8125$ N/mm²

Problem(2):-A flat plate is subjected to a tensile force of 5KN as shown in figure. Take FOS = 2.5, ultimate stress = 200 N/mm^2. Calculate the palte thickness. Assume K_t at sec A-A = 1.8, K_t at sec B-B = 2.16.

Flat plate, tensile force (P) = 5 kN = 5×10^3 N,

$FOS = 2.5;\ \sigma_u = 200\ \text{N/mm}^2$



The stresses are critical at two sections,

1. Section at the fillet *A-A*
2. Section at the hole *B-B*

1. SECTION – *A-A* (Fillet section)

$$D = 45\ ;\ d = 30$$

$$K_t = \frac{\sigma_{max}}{\sigma_o}$$

$$K_t = 1.8$$

σ_o = Nominal stress

$$\sigma_o = \frac{P}{d \times t} = \frac{5 \times 10^3}{30 \times t}$$

$$\sigma_{max} = K_t \cdot \sigma_o$$

$$= \frac{1.8 \times 5 \times 10^3}{30 \times t}$$

$$= \left[\frac{300}{t}\right] \text{N/mm}^2 \ \ldots .\ (1)$$

2. SECTION – *B-B*: (Hole section)

$$\sigma_o = \frac{P}{(W - a)\,t}$$

$$= \frac{5 \times 10^3}{(30 - 15)\,t}$$

$W = 30$ mm

$a = 15$ mm

t = thickness

$K_t = 2.16$

$$\sigma_{max} = K_t \cdot \sigma_o$$

$$= 2.16 \times \frac{5000}{15 \times t}$$

$$\sigma_{max} = \frac{720}{t} \text{ N/mm}^2$$

From (1) and (2) it is seen that maximum stress is induced at the hole section. Equating permissible stress to equation (2)

$$\text{i.e., } \frac{720}{t} = \frac{200}{FOS} = \left(\frac{\sigma_u}{FOS}\right)$$

$$\frac{700}{t} = \frac{200}{2.5}$$

$$\text{or} \quad t = \frac{700 \times 2.5}{200} = 8.75 \text{ mm}$$

$$\therefore t = 8.75 \text{ mm} \quad \text{Take } t = 9 \text{ mm}$$

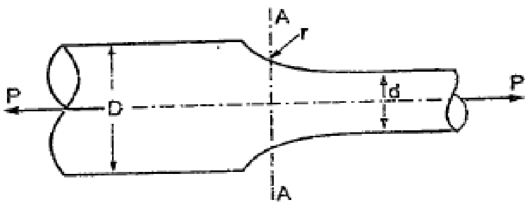
Problem(3):-A stepped shaft has maximum dia = 45mm, minimum dia = 30mm, fillet radius = 6mm, if the shaft is subjected to an axial load of 10KN. Find the maximum stress induced. Assume K_t = 1.45

Given Stepped Shaft

$D = 45$ mm ; $d = 30$ mm

$r = 6$ mm ;

$P = 10 \text{ kN} = 10 \times 10^3 \text{ N}$



$K_t = 1.45$

$$\text{Nominal stress} = \sigma_o = \frac{P}{A} = \frac{10 \times 10^3}{\frac{\pi}{4}(30)^2}$$

$$\sigma_o = 14.147 \text{ N/mm}^2$$

$$K_t = \frac{\sigma_{max}}{\sigma_o}$$

$$K_t \cdot \sigma_o = \sigma_{max}$$

$$\sigma_{max} = \text{Max. stress} = 1.45 \times 14.147 = 20.513 \text{ N/mm}^2$$

$$\sigma_{max} = 20.513 \text{ N/mm}^2$$

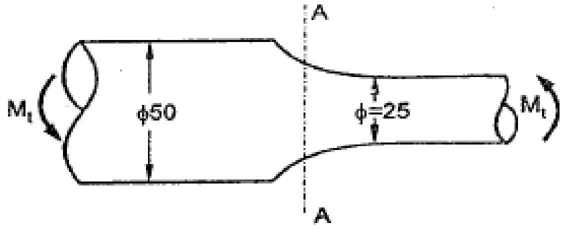
Problem(4):-A stepped shaft has maximum dia =50mm, minimum dia = 25mm, fillet radius 5mm. If the shaft is subjected to a twisting moment of 1500 N-m, find the maximum stress induced. Assume $K_t = 1.35$

Given: Stepped shaft: $D = 50$ mm

$d = 25$ mm

Fillet radius $r = 5$ mm

$T = M_t = 1500$ N–mm



$$J = \frac{\pi}{32} d^4 \qquad K_t = 1.35$$

$$K_t = \frac{\tau_{max}}{\tau_o}$$

$$\tau_o = \text{Nominal shear stress } \tau_o = \frac{M_t \; r_s}{J}$$

where r_s = minimum shaft diameter

$$\frac{d}{2} = \frac{25}{2} = 12.5 \text{ mm}$$

$$= \frac{1500 \times 12.5}{\frac{\pi}{32} \times 25^4} = 0.488 \text{ N/mm}^2$$

$$\therefore K_t = \frac{\tau_{max}}{\tau_o}$$

$$\tau_{max} = 1.35 \times 0.4889 = 0.66 \text{ N/mm}^2$$

Fatigue failure:-

It has been observed that the material fail under fluctuating stresses, at a stress magnitude which is lower than the ultimate strength of the material.

Sometimes, the magnitude is even smaller than the yield strength. The stress lesser than the yield stress is fatigue stress.

"The decreased resistance of the material to fluctuating stresses is called Fatigue failure"

"The failure occuring due to very large number of stress cycles is known as fatigue"

The fatigue failure begins with a crack at some point in the materials. The crack is more likely to occur in the following regions.

1. Regions of discontinuity (such as oil holes, key ways, screw threads etc.)

2. Regions of irregularities in machining operations. (such as scratches on the surface)
3. Internal cracks due to defects in materials (such as blow holes in the casting)

These regions are subjected to stress concentration due to the crack. The crack spreads due to the fluctuating stresses, until the cross-section of the component is too reduced that the remaining portion is subjected to sudden fracture.

Factors affecting fatigue strength:- (what are the factors tobe considered while designing machine parts to avoid fatique failure?)

S.No	Factors affecting fatigue	Description
1	Material factors	This factor includes composition, structure, directional properties and notch sensitivity of the component
2	Manufacturing factors	This factor includes surface finish, heat treatment and residual stresses
3	Geometrical factors	This factor includes size effects and stress concentration
4	Loading factors	This factor includes nature and type of loading
5	Environmental factors	This factor includes corrosion, high temperature and radiation
6	Effect of mean stress	The typical S-N curve for a mean stress is 0. Increasing mean stress decreases fatigue life.

Stress cycle:- (Define stress cycle)

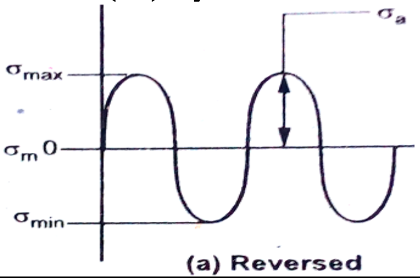
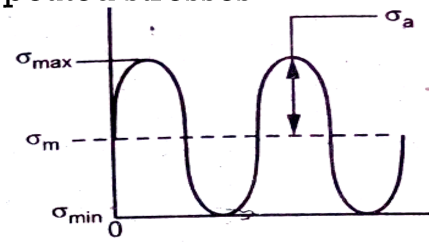
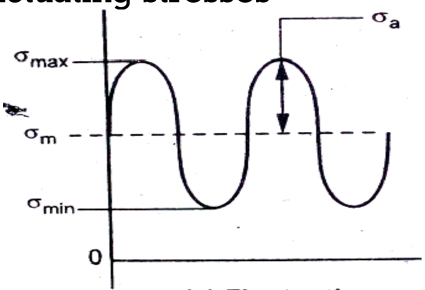
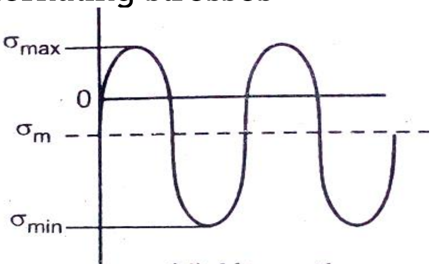
The stress cycles that shows the stress-time relationships are displayed graphically using the sine curve. This diagram indicates maximum, minimum, mean and amplitude stress (Variable stress).

Mean or average stress $\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$

Amplitude or variable stress $\sigma_a = \sigma_v = \frac{\sigma_{max} - \sigma_{min}}{2}$

Stresses due to Fatigue loading:-(write about the design for fluctuating stresses)

S.No	Stresses	Description

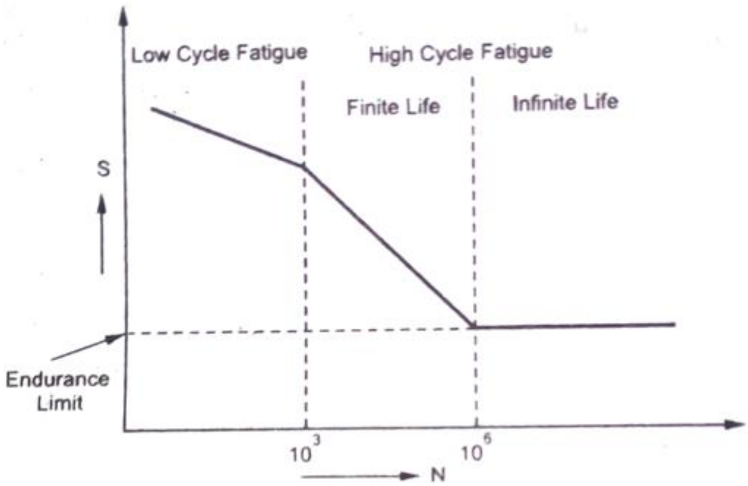
1	Reversed (or) Completely reversed (or) cyclic stresses  (a) Reversed	The stresses which vary from one value of tension to the same value in compression are known as reversed stresses.
2	Repeated stresses  (b) Repeated	The stresses, which vary from zero to a certain maximum value, are known as repeated stresses.
3	Fluctuating stresses  (c) Fluctuating	The stresses vary from maximum value to minimum value of the same nature are known as fluctuating stresses.
4	Alternating stresses  (d) Alternating	The stresses vary from maximum value to minimum value of the opposite nature are known as alternating stresses.

Stress ratio:- Stress ratio is defined as the ratio of the minimum stress to the maximum stress.

$$\text{Stress ratio, } R = \frac{\sigma_{min}}{\sigma_{max}}$$

Fatigue (or) Endurance Limit:-"It is defined as the maximum value of completely reversing stress that the standard specimen can sustain for an infinite number of cycles without fatigue failure"

S-N Curve:-S-N Curve is a graphical representation of stress amplitude (S) versus number of cycles to failure (N) on a logarithmic scale as shown in figure.



Low cycle fatigue: this is mainly applicable for short-lived devices where very large stresses may occur at low cycles. The line unto 10^3 cycles represents the low cycle fatigue.

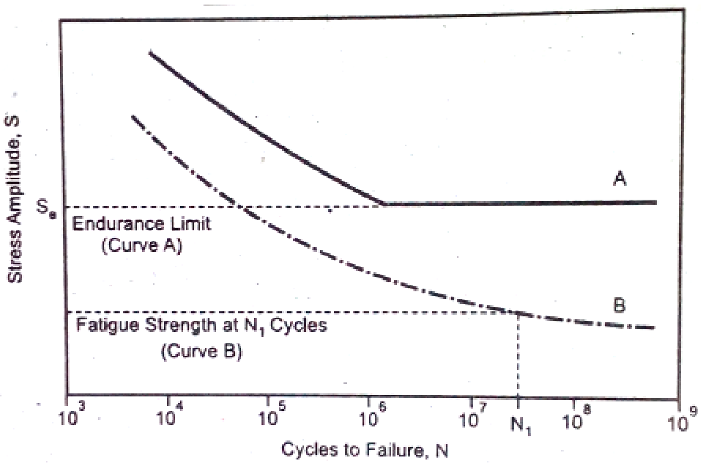
High cycle fatigue with finite life: The line between 10^3 to 10^6 cycles represents the high cycle fatigue with finite life.

Infinite life: If the applied stress level is below the endurance limit of the material, the structure is said to have an infinite life.

High cycle fatigue with infinite life: The line above 10^6 cycles represents the high cycle fatigue with infinite life.

S-N Curve for ferrous and non-ferrous metals:- (Draw S-N curve for mild steel and explain its significance)

The below diagram is also a typical representation of S-N curve for mild steel (ferrous metal) and aluminium (non ferrous metals).



Each test gives one failure point on the S-N curve. These points are scattered on diagram, and an average curve is drawn through the points.

The point at which the S-N curve flattens off is called the endurance limit and the corresponding magnitude of stress is called the endurance strength.

Combined Steady and Variable Stress

The failure points from fatigue tests made with different steels and combinations of mean and variable stresses are plotted in Fig. as functions of variable stress (σ_v) and mean stress (σ_m). The most significant observation is that, in general, the failure point is little related to the mean stress when it is compressive but is very much a function of the mean stress when it is tensile. In practice, this means that fatigue failures are rare when the mean stress is compressive (or negative). Therefore, the greater emphasis must be given to the combination of a variable stress and a steady (or mean) tensile stress.

There are several ways in which problems involving this combination of stresses may be solved, but the following are important from the subject point of view :
1. Gerber method, **2.** Goodman method, and **3.** Soderberg method.

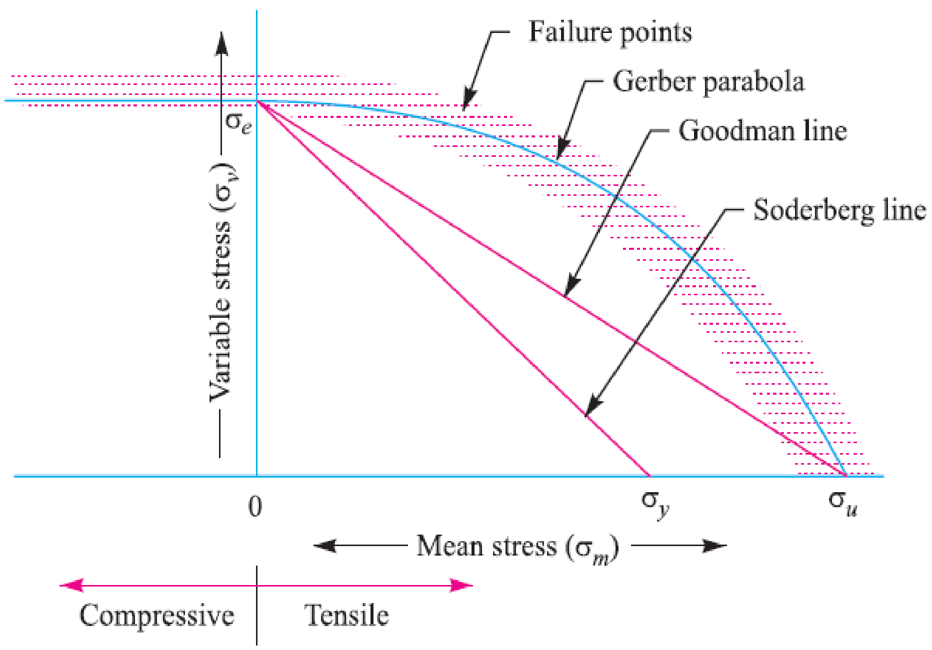


Fig: Combined mean and variable stress.

Gerber Method for Combination of Stresses

according to Gerber relation,

$$\frac{1}{F.S.} = \left(\frac{\sigma_m}{\sigma_u}\right)^2 F.S. + \frac{\sigma_v}{\sigma_e}$$

Considering the fatigue stress concentration factor (K_f), the equation

$$\frac{1}{F.S.} = \left(\frac{\sigma_m}{\sigma_u}\right)^2 F.S. + \frac{\sigma_v \times K_f}{\sigma_e}$$

Goodman Method for Combination of Stresses

A straight line connecting the endurance limit (σ_e) and the ultimate strength (σ_u), as shown by line *AB* in Fig. follows the suggestion of Goodman. A Goodman line is used when the design is based on ultimate strength and may be used for ductile or brittle materials.

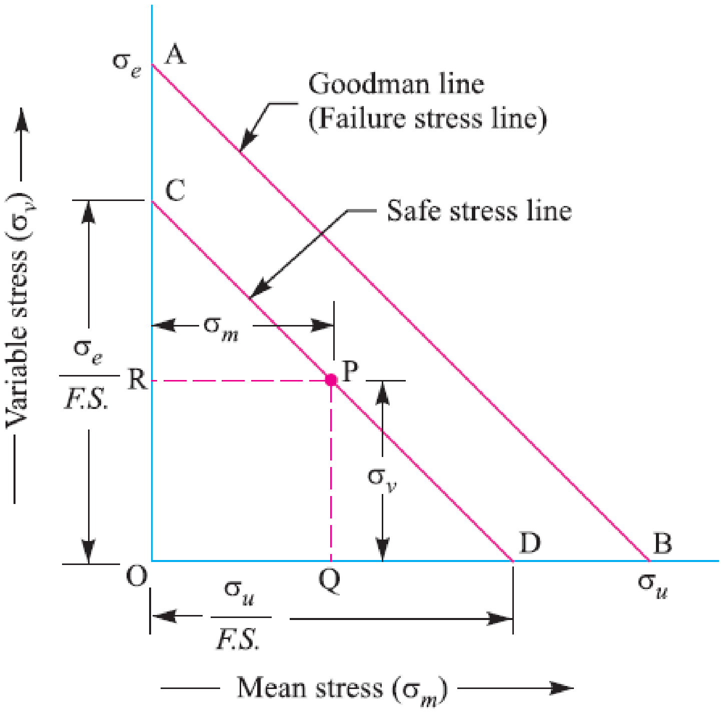


Fig: Goodman method.

1). according to modified Goodman relation,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e}$$

2). according to Goodman's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

...(Taking $K_f = 1$)

...($\because \sigma_{eb} = \sigma_e \times K_b$ and $K_b = 1$)

3). $$\frac{\sigma_v}{\sigma_e/(F.S.)_e} = 1 - \frac{\sigma_m}{\sigma_u/(F.S.)_u}$$

$(F.S.)_e$ = Factor of safety relating to endurance limit, and

$(F.S.)_u$ = Factor of safety relating to ultimate tensile strength.

Soderberg Method for Combination of Stresses

A straight line connecting the endurance limit (σ_e) and the yield strength (σ_y), as shown by the line *AB* in Fig. follows the suggestion of Soderberg line. This line is used when the design is based on yield strength Proceeding in the same way as discussed in Art, the line *AB* connecting σ_e and σ_y, as shown in Fig. is called ***Soderberg's failure stress line***.

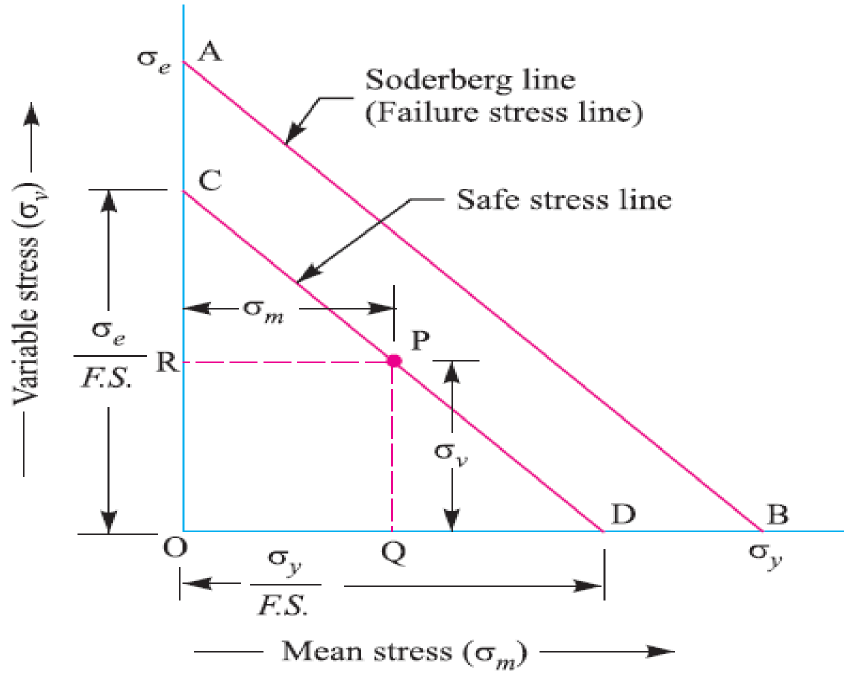


Fig: Soderberg method.

1). According to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

2). according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e}$$

Endurance limit in reversed axial loading, $= \sigma_{ea} = \sigma_e \times K_a$

3). according to Soderberg's formula for reversed axial loading.

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_{ea} \times K_{sur} \times K_{sz}}$$

4). according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}} \qquad \ldots(\text{Taking } K_f = 1)$$

5). according to Soderberg's formula

$$\frac{1}{F.S.} = \frac{\tau_m}{\tau_y} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}}$$

Formulas

mean or average stress, $\sigma_m = \frac{\sigma_1 + \sigma_2}{2}$

variable stress, $\sigma_v = \frac{\sigma_1 - \sigma_2}{2}$

mean or average force, $W_m = \frac{W_{max} + W_{min}}{2}$

Variable force, $W_v = \frac{W_{max} - W_{min}}{2}$

Mean stress, $\sigma_m = \frac{W_m}{A}$

Variable stress, $\sigma_v = \frac{W_v}{A}$

Fatigue stress concentration factor, $K_f = 1 + q\,(K_t - 1)$

K_t = Theoretical stress concentration factor for axial or bending loading, and

K_{ts} = Theoretical stress concentration factor for torsional or shear loading.

$F.S.$ = Factor of safety,

σ_m = Mean stress (tensile or compressive),

σ_u = Ultimate stress (tensile or compressive)

σ_v = Variable stress,

σ_e = Endurance limit for reversal loading.

K_f = Fatigue stress concentration factor.

K_b = Load factor for reversed bending load,

K_{sur} = Surface finish factor, and

K_{sz} = Size factor.

$(F.S.)_e$ = Factor of safety relating to endurance limit, and

$(F.S.)_u$ = Factor of safety relating to ultimate tensile strength.

Combined Variable Normal Stress and Variable Shear Stress

When a machine part is subjected to both variable normal stress and a variable shear stress; then it is designed by using the following two theories of combined stresses : **1.** Maximum shear stress theory, and **2.** Maximum normal stress theory

We have discussed that according to Soderberg's formula,.

Equivalent normal stress due to reversed bending,

$$\sigma_{neb} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fb}}{\sigma_{eb} \times K_{sur} \times K_{sz}}$$

Similarly, equivalent normal stress due to reversed axial loading,

$$\sigma_{nea} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fa}}{\sigma_{ea} \times K_{sur} \times K_{sz}}$$

total equivalent normal stress, $\sigma_{ne} = \sigma_{neb} + \sigma_{nea} = \dfrac{\sigma_y}{F.S.}$

Equivalent shear stress due to reversed torsional or shear loading,

$$\tau_{es} = \tau_m + \frac{\tau_v \times \tau_y \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}}$$

The maximum shear stress theory is used in designing machine parts of ductile materials. According to this theory, maximum equivalent shear stress,

$$\tau_{es(max)} = \frac{1}{2}\sqrt{(\sigma_{ne})^2 + 4\,(\tau_{es})^2} = \frac{\tau_y}{F.S.}$$

The maximum normal stress theory is used in designing machine parts of brittle materials. According to this theory, maximum equivalent normal stress,

$$\sigma_{ne(max)} = \frac{1}{2}(\sigma_{ne}) + \frac{1}{2}\sqrt{(\sigma_{ne})^2 + 4\,(\tau_{es})^2} = \frac{\sigma_y}{F.S.}$$

Problem(1):-subjected to a flexural stress which fluctuates between + 300 MN/m² and – 150 MN/m². Determine the value of minimum ultimate strength according to 1. Gerber relation; 2. Modified Goodman relation; and 3. Soderberg relation. Take yield strength = 0.55 Ultimate strength; Endurance strength = 0.5 Ultimate strength; and factor of safety = 2.

Given data : σ_1 = 300 MN/m² ; σ_2 = – 150 MN/m² ; $\sigma_y = 0.55\,\sigma_u$; $\sigma_e = 0.5\,\sigma_u$; *F.S.* = 2

Let σ_u = Minimum ultimate strength in MN/m².

We know that the mean or average stress,

$$\sigma_m = \frac{\sigma_1 + \sigma_2}{2} = \frac{300 + (-150)}{2} = 75 \text{ MN/m}^2$$

and variable stress,

$$\sigma_v = \frac{\sigma_1 - \sigma_2}{2} = \frac{300 - (-150)}{2} = 225 \text{ MN/m}^2$$

1. *According to Gerber relation*

We know that according to Gerber relation,

$$\frac{1}{F.S.} = \left(\frac{\sigma_m}{\sigma_u}\right)^2 F.S. + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{2} = \left(\frac{75}{\sigma_u}\right)^2 2 + \frac{225}{0.5\sigma_u} = \frac{11\,250}{(\sigma_u)^2} + \frac{450}{\sigma_u} = \frac{11\,250 + 450\,\sigma_u}{(\sigma_u)^2}$$

$$(\sigma_u)^2 = 22\,500 + 900\,\sigma_u$$

$$(\sigma_u)^2 - 900\,\sigma_u - 22\,500 = 0$$

$$\sigma_u = \frac{900 \pm \sqrt{(900)^2 + 4 \times 1 \times 22\,500}}{2 \times 1} = \frac{900 \pm 948.7}{2}$$

$$= 924.35 \text{ MN/m}^2$$

2. *According to modified Goodman relation*

We know that according to modified Goodman relation,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{2} = \frac{75}{\sigma_u} + \frac{225}{0.5\,\sigma_u} = \frac{525}{\sigma_u}$$

$$\sigma_u = 2 \times 525 = 1050 \text{ MN/m}^2$$

3. *According to Soderberg relation*

We know that according to Soderberg relation,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{2} = \frac{75}{0.55\,\sigma_u} + \frac{255}{0.5\,\sigma_u} = \frac{586.36}{\sigma_u}$$

$$\sigma_u = 2 \times 586.36 = 1172.72 \text{ MN/m}^2$$

Problem(2):-A bar of circular cross-section is subjected to alternating tensile forces varying from a minimum of 200 kN to a maximum of 500 kN. It is to be manufactured of a material with an ultimate tensile strength of 900 MPa and an endurance limit of 700 MPa. Determine the diameter of bar using safety factors of 3.5 related to ultimate tensile strength and 4 related to endurance limit and a stress concentration factor of 1.65 for fatigue load. Use Goodman straight line as basis for design.

Solution. Given : W_{min} = 200 kN ; W_{max} = 500 kN ; σ_u = 900 MPa = 900 N/mm^2 ; σ_e = 700 MPa = 700 N/mm^2 ; $(F.S.)_u$ = 3.5 ; $(F.S.)_e$ = 4 ; K_f = 1.65

$$d = \text{Diameter of bar in mm.}$$

$$\text{Area, } A = \frac{\pi}{4} \times d^2 = 0.7854\, d^2 \text{ mm}^2$$

We know that mean or average force,

$$W_m = \frac{W_{max} + W_{min}}{2} = \frac{500 + 200}{2} = 350 \text{ kN} = 350 \times 10^3 \text{ N}$$

$$\text{Mean stress, } \sigma_m = \frac{W_m}{A} = \frac{350 \times 10^3}{0.7854\, d^2} = \frac{446 \times 10^3}{d^2} \text{ N/mm}^2$$

$$\text{Variable force, } W_v = \frac{W_{max} - W_{min}}{2} = \frac{500 - 200}{2} = 150 \text{ kN} = 150 \times 10^3 \text{ N}$$

$$\text{Variable stress, } \sigma_v = \frac{W_v}{A} = \frac{150 \times 10^3}{0.7854\, d^2} = \frac{191 \times 10^3}{d^2} \text{ N/mm}^2$$

We know that according to Goodman's formula,

$$\frac{\sigma_v}{\sigma_e / (F.S.)_e} = 1 - \frac{\sigma_m . K_f}{\sigma_u / (F.S.)_u}$$

$$\frac{\dfrac{191 \times 10^3}{d^2}}{700/4} = 1 - \frac{\dfrac{446 \times 10^3}{d^2} \times 1.65}{900/3.5}$$

$$\frac{1100}{d^2} = 1 - \frac{2860}{d^2} \quad \text{or} \quad \frac{1100 + 2860}{d^2} = 1$$

$$d^2 = 3960 \quad \text{or} \quad d = 62.9 \text{ say } 63 \text{ mm} \quad \textbf{Ans.}$$

<u>Problem(3):-</u>Determine the thickness of a 120 mm wide uniform plate for safe continuous operation if the plate is to be subjected to a tensile load that has a maximum value of 250kN and a minimum value of 100kN. The properties of the plate material are as follows:
Endurance limit stress = 225MPa, and Yield point stress = 300MPa.The factor of safety based on yield point may be taken as 1.5.

Given : b = 120 mm ; W_{max} = 250 kN; W_{min} = 100 kN ; σ_e = 225 MPa = 225 N/mm^2 ; σ_y = 300 MPa = 300 N/mm^2; $F.S.$ = 1.5

Let t = Thickness of the plate in mm.

∴ Area, $A = b \times t = 120\, t$ mm^2

We know that mean or average load,

$$W_m = \frac{W_{max} + W_{min}}{2} = \frac{250 + 100}{2} = 175 \text{ kN} = 175 \times 10^3 \text{ N}$$

$$\text{Mean stress, } \sigma_m = \frac{W_m}{A} = \frac{175 \times 10^3}{120\,t} \text{ N/mm}^2$$

$$\text{Variable load, } W_v = \frac{W_{max} - W_{min}}{2} = \frac{250 - 100}{2} = 75 \text{ kN} = 75 \times 10^3 \text{ N}$$

$$\text{Variable stress, } \sigma_v = \frac{W_v}{A} = \frac{75 \times 10^3}{120\,t} \text{ N/mm}^2$$

According to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{1.5} = \frac{175 \times 10^3}{120\,t \times 300} + \frac{75 \times 10^3}{120\,t \times 225} = \frac{4.86}{t} + \frac{2.78}{t} = \frac{7.64}{t}$$

$$t = 7.64 \times 1.5 = 11.46 \text{ say } 11.5 \text{ mm} \quad \textbf{Ans.}$$

Problem(4):-A steel rod is subjected to a reversed axial load of 180kN. Find the diameter of the rod for a factor of safety of 2. Neglect column action. The material has an ultimate tensile strength of 1070MPa and yield strength of 910MPa. The endurance limit in reversed bending may be assumed to be one-half of the ultimate tensile strength. Other correction factors may be taken as follows:
For axial loading = 0.7; For machined surface = 0.8 ; For size = 0.85 ; For stress concentration = 1.0.

Given data: W_{max} = 180 kN ; W_{min} = – 180 kN ; $F.S.$ = 2 ; σ_u = 1070 MPa = 1070 N/mm^2; σ_y = 910 MPa = 910 N/mm^2 ; $\sigma_e = 0.5\ \sigma_u$; K_a = 0.7 ; K_{sur} = 0.8 ; K_{sz} = 0.85 ; K_f = 1

Let d = Diameter of the rod in mm.

$$\therefore \quad \text{Area, } A = \frac{\pi}{4} \times d^2 = 0.7854\, d^2 \text{ mm}^2$$

We know that the mean or average load,

$$W_m = \frac{W_{max} + W_{min}}{2} = \frac{180 + (-180)}{2} = 0$$

$\therefore$ Mean stress, $\sigma_m = \frac{W_m}{A} = 0$

Variable load, $W_v = \frac{W_{max} - W_{min}}{2} = \frac{180 - (-180)}{2} = 180 \text{ kN} = 180 \times 10^3 \text{ N}$

$\therefore$ Variable stress, $\sigma_v = \frac{W_v}{A} = \frac{180 \times 10^3}{0.7854\, d^2} = \frac{229 \times 10^3}{d^2} \text{ N/mm}^2$

Endurance limit in reversed axial loading,

$$\sigma_{ea} = \sigma_e \times K_a = 0.5\, \sigma_u \times 0.7 = 0.35\, \sigma_u \quad ...(\because\ \sigma_e = 0.5\, \sigma_u)$$
$$= 0.35 \times 1070 = 374.5 \text{ N/mm}^2$$

We know that according to Soderberg's formula for reversed axial loading,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_{ea} \times K_{sur} \times K_{sz}}$$

$$\frac{1}{2} = 0 + \frac{229 \times 10^3 \times 1}{d^2 \times 374.5 \times 0.8 \times 0.85} = \frac{900}{d^2}$$

$d^2 = 900 \times 2 = 1800$ or $d = 42.4$ mm **Ans.**

Problem(5):-A circular bar of 500 mm length is supported freely at its two ends. It is acted upon by a central concentrated cyclic load having a minimum value of 20 kN and a maximum value of 50 kN. Determine the diameter of bar by taking a factor of safety of 1.5, size effect of 0.85, surface finish factor of 0.9. The material properties of bar are given by : ultimate strength of 650 MPa, yield strength of 500 MPa and endurance strength of 350 MPa.

Given data: l = 500 mm ; W_{min} = 20 kN = 20 × 10^3 N ; W_{max} = 50 kN = 50 × 10^3 N ; $F.S.$ = 1.5 ; K_{sz} = 0.85 ; K_{sur} = 0.9 ; σ_u = 650 MPa = 650 N/mm^2 ; σ_y = 500 MPa = 500 N/mm^2 ; σ_e = 350 MPa = 350 N/mm2

Let d = Diameter of the bar in mm.

We know that the maximum bending moment,

$$M_{max} = \frac{W_{max} \times l}{4} = \frac{50 \times 10^3 \times 500}{4} = 6250 \times 10^3 \text{ N-mm}$$

and minimum bending moment,

$$M_{min} = \frac{W_{min} \times l}{4} = \frac{20 \times 10^3 \times 500}{4} = 2550 \times 10^3 \text{ N-mm}$$

∴ Mean or average bending moment,

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{6250 \times 10^3 + 2500 \times 10^3}{2} = 4375 \times 10^3 \text{ N-mm}$$

and variable bending moment,

$$M_v = \frac{M_{max} - M_{min}}{2} = \frac{6250 \times 10^3 - 2500 \times 10^3}{2} = 1875 \times 10^3 \text{ N-mm}$$

Section modulus of the bar,

$$Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3 \text{ mm}^3$$

∴ Mean or average bending stress,

$$\sigma_m = \frac{M_m}{Z} = \frac{4375 \times 10^3}{0.0982\, d^3} = \frac{44.5 \times 10^6}{d^3} \text{ N/mm}^2$$

and variable bending stress,

$$\sigma_v = \frac{M_v}{Z} = \frac{1875 \times 10^3}{0.0982\, d^3} = \frac{19.1 \times 10^6}{d^3} \text{ N/mm}^2$$

We know that according to Goodman's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.5} = \frac{44.5 \times 10^6}{d^3 \times 650} + \frac{19.1 \times 10^6 \times 1}{d^3 \times 350 \times 0.9 \times 0.85} \quad \text{...(Taking } K_f = 1\text{)}$$

$$= \frac{68 \times 10^3}{d^3} + \frac{71 \times 10^3}{d^3} = \frac{139 \times 10^3}{d^3}$$

$d^3 = 139 \times 10^3 \times 1.5 = 209 \times 10^3$ or $d = 59.3$ mm

and according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

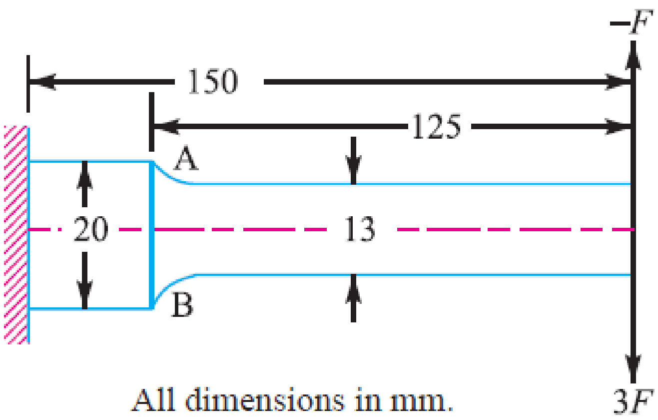
$$\frac{1}{1.5} = \frac{44.5 \times 10^6}{d^3 \times 500} + \frac{19.1 \times 10^6 \times 1}{d^3 \times 350 \times 0.9 \times 0.85} \quad \text{...(Taking } K_f = 1\text{)}$$

$$= \frac{89 \times 10^3}{d^3} + \frac{71 \times 10^3}{d^3} = \frac{160 \times 10^3}{d^3}$$

$$d^3 = 160 \times 10^3 \times 1.5 = 240 \times 10^3 \quad \text{or} \quad d = 62.1 \text{ mm}$$

Taking larger of the two values, we have d = 62.1 mm

<u>Problem(6):-</u>A cantilever beam made of cold drawn carbon steel of circular cross-section as shown in Fig., is subjected to a load which varies from – F to 3 F. Determine the maximum load that this member can withstand for an indefinite life using a factor of safety as 2. The theoretical stress concentration factor is 1.42 and the notch sensitivity is 0.9. Assume the following values :
Ultimate stress = 550 MPa; Yield stress = 470 MPa; Endurance limit = 275 MPa
Size factor = 0.85;Surface finish factor= 0.89



Given data : W_{min} = – F ; W_{max} = 3 F ; $F.S.$ = 2 ; K_t = 1.42 ; q = 0.9 ; σ_u = 550 MPa = 550 N/mm² ; σ_y = 470 MPa = 470 N/mm² ; σ_e = 275 MPa = 275 N/mm² ; K_{sz} = 0.85 ; K_{sur} = 0.89

We know that maximum bending moment at point A,

$$M_{max} = W_{max} \times 125 = 3F \times 125 = 375\, F \text{ N-mm}$$

and minimum bending moment at point A,

$$M_{min} = W_{min} \times 125 = -F \times 125 = -125\, F \text{ N-mm}$$

∴ Mean or average bending moment,

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{375\,F + (-125\,F)}{2} = 125\,F \text{ N-mm}$$

and variable bending moment,

$$M_v = \frac{M_{max} - M_{min}}{2} = \frac{375\ F - (-125\ F)}{2} = 250\,F \text{ N-mm}$$

Section modulus, $$Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32}(13)^3 = 215.7 \text{ mm}^3 \quad ...(\because d = 13 \text{ mm})$$

$\therefore$ Mean bending stress, $$\sigma_m = \frac{M_m}{Z} = \frac{125\ F}{215.7} = 0.58\ F \text{ N/mm}^2$$

and variable bending stress, $$\sigma_v = \frac{M_v}{Z} = \frac{250\ F}{215.7} = 1.16\ F \text{ N/mm}^2$$

Fatigue stress concentration factor, $K_f = 1 + q\,(K_t - 1) = 1 + 0.9\,(1.42 - 1) = 1.378$

We know that according to Goodman's formula

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{2} = \frac{0.58\ F}{550} + \frac{1.16\,F \times 1.378}{275 \times 0.89 \times 0.85}$$

$$= 0.001\ 05\ F + 0.007\ 68\ F = 0.008\ 73\ F$$

$\therefore$ $$F = \frac{1}{2 \times 0.008\,73} = 57.3 \text{ N}$$

and according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{2} = \frac{0.58\ F}{470} + \frac{1.16\,F \times 1.378}{275 \times 0.89 \times 0.85}$$

$$= 0.001\ 23\ F + 0.007\ 68\ F = 0.008\ 91\ F$$

$$F = \frac{1}{2 \times 0.008\ 91} = 56 \text{ N}$$

Taking larger of the two values, we have $F = 57.3$ N

<u>Problem(7):-</u>A simply supported beam has a concentrated load at the centre which fluctuates from a value of P to 4 P. The span of the beam is 500 mm and its cross-section is circular with a diameter of 60 mm. Taking for the beam material an ultimate stress of 700 MPa, a yield stress of 500MPa, endurance limit of 330MPa for reversed bending, and a factor of safety of 1.3, calculate the maximum value of P. Take a size factor of 0.85 and a surface finish factor of 0.9.

Given data : $W_{min} = P$; $W_{max} = 4P$; $L = 500$ mm; $d = 60$ mm ; $\sigma_u = 700$ MPa =

700 N/mm^2 ; σ_y = 500 MPa = 500 N/mm^2 ; σ_e = 330 MPa = 330 N/mm^2 ; *F.S.* = 1.3 ; K_{sz} = 0.85 ; K_{sur} = 0.9

We know that maximum bending moment,

$$M_{max} = \frac{W_{max} \times L}{4} = \frac{4P \times 500}{4} = 500\,P \text{ N-mm}$$

and minimum bending moment,

$$M_{min} = \frac{W_{min} \times L}{4} = \frac{P \times 500}{4} = 125\,P \text{ N-mm}$$

∴ Mean or average bending moment,

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{500\,P + 125\,P}{2} = 312.5\,P \text{ N-mm}$$

and variable bending moment,

$$M_v = \frac{M_{max} - M_{min}}{2} = \frac{500\,P - 125\,P}{2} = 187.5\,P \text{ N-mm}$$

Section modulus, $$Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32}(60)^3 = 21.21 \times 10^3 \text{ mm}^3$$

∴ Mean bending stress,

$$\sigma_m = \frac{M_m}{Z} = \frac{312.5\,P}{21.21 \times 10^3} = 0.0147\,P \text{ N/mm}^2$$

and variable bending stress,

$$\sigma_v = \frac{M_v}{Z} = \frac{187.5\,P}{21.21 \times 10^3} = 0.0088\,P \text{ N/mm}^2$$

We know that according to Goodman's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.3} = \frac{0.0147\,P}{700} + \frac{0.0088\,P \times 1}{330 \times 0.9 \times 0.85}$$ ...(Taking K_f = 1)

$$= \frac{21P}{10^6} + \frac{34.8P}{10^6} = \frac{55.8P}{10^6}$$

$$P = \frac{1}{1.3} \times \frac{10^6}{55.8} = 13\ 785 \text{ N} = 13.785 \text{ kN}$$

and according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.3} = \frac{0.0147P}{500} + \frac{0.0088P \times 1}{330 \times 0.9 \times 0.85} = \frac{29.4P}{10^6} + \frac{34.8P}{10^6} = \frac{64.2P}{10^6}$$

$$P = \frac{1}{1.3} \times \frac{10^6}{64.2} = 11\ 982 \text{ N} = 11.982 \text{ kN}$$

From the above, we find that maximum value of P = 13.785 kN